Nuclear energy – NAKE-12
Final exam
30-10-2018 18:30 – 21:30

Please write your name and student number on each sheet of paper you hand in.

Problem 1 – Neutron poisons (14 pts)

The fission process produces a variety of fission fragments, of which some are neutron absorbers called neutron poisons. One of the most important neutron poisons in a reactor is ^{135}Xe. Xe-135 is produced directly by fission and indirectly by the decay of I-135. The decay half-life of ^{135}I is shorter than that of ^{135}Xe.

The concentration of Iodine (I) as a function of time is
$\frac{dI}{dt} = \gamma_I \, \Sigma_f^{fuel} \phi - \lambda_I \, I.$

a. Write the equilibrium concentration of I. (2 pts)

At equilibrium: $\frac{dI}{dt} = 0 \longrightarrow \gamma_I \, \Sigma_f^{fuel} \phi - \lambda_I \, I = 0 \longrightarrow I = \frac{\gamma_I \, \Sigma_f^{fuel} \phi}{\lambda_I}$

b. Write the expression of the rate of change in concentration for Xenon (Xe). (4 pts)

$$\frac{dXe}{dt} = \gamma_{Xe} \, \Sigma_f^{fuel} \phi + \lambda_I \, I - \lambda_{Xe} \, Xe - Xe \, \sigma_{cap}^{Xe} \, \phi$$

c. Calculate the equilibrium concentration of Xenon. (4 pts)

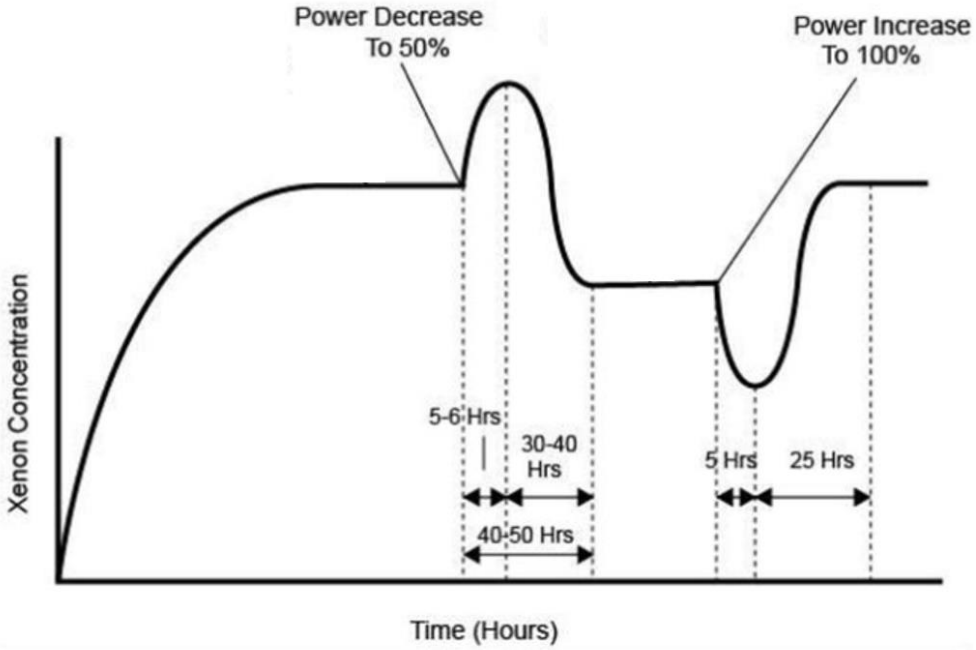
At equilibrium: $\frac{dXe}{dt} = 0; \; \gamma_{Xe} \, \Sigma_f^{fuel} \phi + \lambda_I \, I - \lambda_{Xe} \, Xe - Xe \, \sigma_{cap}^{Xe} \, \phi = 0$

$$Xe = \frac{\gamma_{Xe} \, \Sigma_f^{fuel} \phi + \lambda_I \, I}{\lambda_{Xe} + \sigma_{cap}^{Xe} \, \phi} = \frac{(\gamma_{Xe} + \gamma_I) \, \Sigma_f^{fuel} \phi}{\lambda_{Xe} + \sigma_{cap}^{Xe} \, \phi}$$

$$Xe = \frac{(0.0025 + 0.0638) \times 0.002 \times 6 \; 10^{14}}{2.1 \; 10^{-5} + 3.2 \; 10^{-18} \times 6 \; 10^{14}} = 4.1 \; 10^{13} atoms/cm^3$$

λ_{Xe}= *2.10*10^{-5} s^{-1}*
γ_I = 0.0638
γ_{Xe} = 0.0025
$\sigma_{capture}^{Xe}$= 3.20*10^{-18} cm^2
Σ_f^{fuel}= 0.002 cm^{-1}
$\phi = 6 \times 10^{14} cm^{-2} s^{-1}$

d. The figure below shows the Xenon concentration as a function of time. Explain the behaviour of the concentration as a function of time and changes in power. (4 pts)



As the reactor start, the Xenon population is zero. With the burning of the fuel, the Xenon population increases and reaches equilibrium. When the power decreases by 50%, the neutron flux is reduced by the same value and the concentration of Xenon increases due to the faster decay of Iodine. The Xenon concentration reaches a new (lower) equilibrium value. As the power is increased back to 100%, the neutron flux is increased by the same value and the burnup of Xenon becomes dominant, as seen by the dip in concentration. The Xenon population increases to its original equilibrium concentration.

Problem 2 – Fusion (19 pts)

Part I

Fusion in the Sun proceeds through so-called proton-proton chains. The fusion reactions in the second p-p chain are:

$$p + p \longrightarrow d + e^+ + \nu \quad with\ E_\nu = 0.265\ MeV$$

$$p + d \longrightarrow {}^3_2He + \gamma$$

$${}^3_2He + {}^4_2He \longrightarrow {}^7_4Be + \gamma$$

$${}^7_4Be + e^- \longrightarrow {}^7_3Li + \nu \quad with\ E_\nu = 0.860\ MeV$$

$${}^7_3Li + p \longrightarrow {}^4_2He + {}^4_2He$$

Element	**Excess mass (keV)**	**Atomic mass (u)**
Electron	511	5.489×10^{-4}
H	7288.97061	1.007825032
^{2}H (d)	13135.72176	2.014101778
^{3}He	14931.21793	3.016029322
^{4}He	2424.91561	4.002603254
^{7}Li	14907.105	7.016003437
^{7}Be	15769.00	7.016928720

a. Calculate the total energy produced in the second proton-proton chain. (3 pts)

Q(p+p) = 2 x 7288.97061 – (13135.72176 + 2 x 511) = 0.420 MeV

Q(p+d) = 7288.97061 + 13135.72176 – 14931.21793 = 5.493 MeV

Q(^{3}He+^{4}He) = 14931.21793 + 2424.91561 – 15769.00 = 1.587 MeV

Q(^{7}Be+e$^-$) = 15769.00 – 14907.105 = 0.869 MeV

Q(^{7}Li+p) = 14907.105 + 7288.97061 – 2 x 2424.91561 = 17.346 MeV

The total energy produced in the second p-p chain is the sum of the individual Q-values.

Q_{tot} = 25.715 MeV

b. How much energy is released in the Sun? (3 pts)

The two neutrinos escape the Sun without depositing any energy. The positron annihilates with an electron in the Sun and produces two photons of 511 keV each.

Q_{Sun} = Q_{tot} – 0.265 – 0.86 + 2 x 0.511 = 25.61 MeV

Part II

Fusion power is a form of power generation, in which the energy generated by nuclear fusion reactions will be used to produce electricity. In fusion reactors, the heat production is based on the $d + t \longrightarrow {}^4_2He + n$ reaction.

a. Calculate the energy released by the reaction. (2 pts)

Element	Atomic mass (u)
n	1.0086649158
^{2}H (d)	2.014101778
^{3}H (t)	3.01604928199
^{4}He	4.002603254

The energy released by the reaction is the Q-value.
Q = 2.014101778 + 3.01604928199 – (4.002603254 + 1.0086649158) = 0.01888289019 u
Q = 0.01888289019 x 931.494 = 17.59 MeV

b. Which of the two outgoing particles (^{4}He and n) is used to generate heat and ultimately electricity? (2 pts)

The lighter particle is most likely to escape the vacuum vessel, therefore the neutron is used to generate heat. Moreover, the neutron is not confined by the magnetic fields.

c. Calculate the energy of this particle. (4 pts)

Using the conservation of energy and momentum, we find:

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = Q$$

$$m_A v_A + m_B v_B = 0$$

$E_A = \frac{1}{2}m_A v_A^2 = \frac{m_B}{m_A+m_B}Q$; $E_B = \frac{1}{2}m_B v_B^2 = \frac{m_B}{m_A+m_B}Q$

$$E_\alpha = 3.54\ MeV;\ E_n = 14.06\ MeV$$

Part III

a. Give the definition of the fusion energy gain factor Q, and explain the term breakeven. (2 pts)

The fusion energy gain factor is the ratio of fusion power produced to the power required to maintain plasma in steady state. Breakeven is the situation when Q = 1.

b. What will the roles of the blanket surrounding the vacuum vessel be in the fusion reactor ITER? (3 pts)

The roles of the blanket in ITER are threefold:

- The heat (kinetic energy) deposited by the neutron in the blanket is collected by coolant flowing through it.
- Tritium breeding (from lithium)
- Shielding: the blanket (~ 1 m) thick enough to slow down the neutrons

Problem 3 – Reactivity (12 pts)

Consider an under-moderated pressurized water reactor.

a. Explain what happens when the temperature of the fuel increases and the effect on the reactivity of the core. (4 pts)

If the fuel temperature increases, the relative thermal motion of atoms in the fuel and the neutrons increases and causes a broadening of the resonances in the U-238 capture cross section (Doppler broadening). More neutrons are captured in the resonances and the reactivity decreases (reduction of resonance escape probability p).

b. Which factors of the effective multiplication factor are the most affected by a rise in the moderator temperature? Justify your answer. (4 pts)

The increase of the moderator temperature causes the energy of thermal neutrons to shift to higher energy because of thermal expansion (neutrons travel further before interacting/scattering with/off a moderator atom). The fission cross section is reduced. The fast and thermal non-leakage probabilities decrease. The resonance capture increases so the resonance escape probability p decreases. The reduction in atomic density lowers the macroscopic absorption cross-sections in the moderator, and therefore, increases the thermal utilization (f).

c. Explain how a reactor such as a PWR is inherently safe. (4 pts)

In an under-moderated reactor, the increase in reactivity causes the reactor to produce more power, which raises the temperature of the core and adds negative reactivity, which in turn slows down the power rise.

Problem 4 – ^{210}Po (5 pts)

Polonium-210 has a half-life of 138.376 days. Calculate the activity of 1 mg of ^{210}Po in Curie.

$$\lambda = \frac{Ln(2)}{T_{1/2}} = \frac{Ln(2)}{138.376 \times 24 \times 3600} = 5.8\ 10^{-8} s^{-1}$$

$$N = m\frac{N_A}{A} = 10^{-3}\,\frac{6.022\ 10^{23}}{210} = 2.87\ 10^{18}\ atoms$$

$$A = \lambda\, N = 1.66\ 10^{11}\ Bq = 4.5\ Ci$$